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RAPID COMMUNICATION

Finger Coordination Under Artificial Changes in Finger Strength Feedback: A Study Using Analytical Inverse Optimization

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ABSTRACT. A recently developed method of analytical inverse optimization (ANIO) was used to compute cost functions based on sets of experimental observations in 4-finger pressing tasks with accurate total force and moment production. In different series, feedback on total force and moment was provided using the index finger force at its value, doubled, or halved. Finger force data across different force–moment combinations formed a plane. This allowed reconstructing cost functions as 2nd-order polynomials with linear terms. Changes in the coefficients of the cost function across the 3 series allowed the authors to offer a biomechanical interpretation related to constraints on finger forces with different lever arms. ANIO allows the authors to describe preferred regions within the space of solutions for redundant tasks in terms of cost functions.

Keywords: ANIO approach, finger, force, redundancy, inverse optimization

In multidigit tasks, the number of task constraints is typically smaller than the number of fingers (Zatsiorsky & Latash, 2008). As a result, an infinite number of finger force combinations can be used to solve any task. This is an example of the problem of motor redundancy (Bernstein, 1967). One commonly used approach to such problems is optimization: It is assumed that the controller selects from an infinite set a solution that minimizes a particular cost function. Most studies applied this method using cost functions that represented educated guesses by the researchers (e.g., Nelson, 1983; Prilutsky, 2000; Rosenbaum, Meulenbroek, Vaughan, & Jansen, 2001).

Recently, a new method was introduced, Analytical Inverse Optimization (ANIO), which allows reconstructing a cost function based on a set of experimental observations (Terekhov, Pesin, Niu, Latash, & Zatsiorsky, 2010), not on the researcher's intuition. In a previous study, we applied this method to four-finger pressing tasks that required accurate production of various combinations of the total force (F_{TOT}) and total moment of force (M_{TOT}); the same ranges of M_{TOT} for each F_{TOT} were used (Park, Zatsiorsky, & Latash, 2010). The reconstructed cost function was a second-order polynomial with essentially nonzero linear terms. The coefficients at the second-order terms were all positive, larger for the index and little fingers. The coefficients at the first-order terms for the index and little fingers were negative, whereas they were positive for the middle and ring fingers.

High positive coefficients in an additive cost function imply that the use of that finger force is discouraged (i.e., smaller finger forces can be used to arrive at the same value of the cost function). Negative coefficients imply that the use of that finger force is encouraged because applying more force by that finger decreases the cost function value. In a cost

function that includes the quadratic and linear terms, the linear terms would dominate at relatively low forces while quadratic terms would dominate at high forces. Because the same range of M_{TOT} was used for different F_{TOT} , at low F_{TOT} using fingers with large lever arms (index and little) was necessary for high M_{TOT} values. In contrast, at high F_{TOT} , using fingers with large lever arms could lead to excessive M_{TOT} . Then, fingers producing moment of force in the opposite direction may have to produce considerable force (moment antagonist; see Zatsiorsky, Gregory, & Latash, 2002), which is a wasteful strategy.

To test whether finger coordination is driven by this relatively simple biomechanical interpretation, we performed a study in which the subjects performed identical sets of tasks (the same $\{F_{TOT}; M_{TOT}\}$ combinations) while the index finger force was used to compute F_{TOT} and M_{TOT} either using the actual force (veridical) or the force multiplied by two (twice as strong, $2F_I$), or the force divided by two (half as strong, $0.5F_I$; a manipulation similar to the one used in Latash, Gelfand, Li, & Zatsiorsky, 1998). We hypothesized that (a) the ANIO method would be able to reconstruct cost functions in all conditions; (b) the pattern of the cost function (a quadratic function with linear terms) would be preserved, and only the coefficients would vary; (c) in the $2F_I$ condition, the coefficients at quadratic terms for the middle, ring, and little fingers would be reduced (their use is encouraged as compared to the index finger), whereas they would be increased in the $0.5F_I$ condition; and (d) the coefficients at linear terms for the index and little fingers (with large lever arms) would be relatively increased in the $2F_I$ condition and reduced in the $0.5F_I$ condition.

Method

Subjects

Seven right-handed male subjects took part in the study. Their mean age was 29.86 ± 2.41 years, their mean height was 177.14 ± 6.26 cm, and their mean weight was 69.71 ± 7.39 kg. All subjects were healthy, without a previous history of neuropathies or traumas to their upper extremities. All subjects signed a consent form according to the procedures approved by the Office for Research Protection of the Pennsylvania State University.

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Apparatus

Four force sensors (Nano-17, ATI Industrial Automation, Garner, NC) were used to measure finger pressing forces (i.e., normal forces). The sensors were placed on the panel (140 × 90 × 5 mm) with four slots, which allowed adjusting sensor positions according to the individual hand and finger lengths of each subject. The distance between the slots was 3.0 cm in the medialateral direction. The panel was mechanically fixed to the immovable table. The sampling frequency was set at 200 Hz.

Procedures

The subjects sat in a chair facing the computer screen and positioned their right upper arm on a wrist-forearm brace that was fixed to the table. The forearm was held stationary with Velcro straps. The subjects placed the right hand fingertips on the sensor centers and kept the fingertips in contact with the sensors at all times. A wooden piece was placed underneath the subject's right palm to ensure a constant configuration of the hand and fingers. The subjects were free to select a comfortable position of the thumb. The experiment consisted of two maximal voluntary contraction (MVC) tasks and two sessions of force-moment production tasks.

The MVC tasks included four-finger maximal voluntary contraction (MVC_{IMRL}) and index finger MVC (MVC_I) tasks. During each task, the subjects were instructed to produce maximal force by either all four fingers or the index finger only within 3 s. The peak force during this time interval was measured and used to determine target force and moment magnitudes in the following force-moment production tasks. For the index finger MVC (MVC_I) task, the subjects were asked to keep all the fingers on the sensors, while not paying attention to possible force productions by the other fingers of the hand.

Session 1: Force–Moment Production

The subjects were asked to produce various combinations of steady-state levels of total normal force (F_{TOT}) and moment of normal force (M_{TOT}) simultaneously as accurately as possible; M_{TOT} into pronation (PR) or supination (SU) was computed as a linear function of normal finger forces multiplied by the lever arms with respect to the midpoint between the middle and ring fingers. F_{TOT} and M_{TOT} were displayed on the computer screen along the vertical and horizontal axis, respectively. The subjects were given 4 s to reach the target values of F_{TOT} and M_{TOT} as accurately as possible and maintain these values for at least 1.5 s. The force target levels included 20, 25, 30, 35, 40, 45, 50, 55, and 60% of MVC_{IMRL} (9 levels). The moment target levels included 2.0PR, 1.5PR, 1.0PR, 0.5PR, 0PR, 0.5SU, 1.0SU, 1.5SU, and 2SU (9 levels). The product of 7% of MVC_I by the lever arm of the index finger ($d_i = 4.5$ cm) was taken as a unit of M_{TOT} (1PR or –1SU). Each subject performed a total of 81

trials (9 levels of forces × 9 levels of moments × 1 trial = 81 trials) during Session 1.

Session 2: Force–Moment Production With Scaled Feedback

Session 2 was identical to Session 1, with only slight modification. The computed values of F_{TOT} and M_{TOT} were artificially computed using the index finger force multiplied by 2 ($2F_I$) or divided by 2 ($0.5F_I$). As in Session 1, there were 81 experimental conditions for each of the scaled feedback conditions. Thus, each subject performed a total of 162 trials during Session 2. Before starting Session 2, the subjects were explicitly told how the force-moment feedback was going to be distorted. They had sufficient practice trials (about 15–20 trials over 10 min of practice for each of the $2F_I$ and $0.5F_I$ conditions) to be familiarized with the scaled feedback condition. A 30-s break was given between trials to avoid fatigue throughout the experiment. The order of $\{F_{TOT}, M_{TOT}\}$ combinations was randomized. For Sessions 1 and 2, average F_{TOT} and M_{TOT} over 1.5 s in the middle of the 4 s interval were displayed immediately at the end of each trial to check the performance error from the prescribed values. If the performance error of either F_{TOT} or M_{TOT} exceeded the criteria— $(\sqrt{(F_{TOT} - F_{Target})^2} > 0.02 \times MVC_{IMRL}, \sqrt{(M_{TOT} - M_{Target})^2} > 0.2 \times 1SU)$ —the trial was rejected and performed again. This happened in 26 out of a total of 1701 trials across all subjects and conditions.

Data Analysis

The data were digitally low-pass filtered with a zero-lag, fourth-order Butterworth filter at 5 Hz. The actual finger forces, not scaled forces, were used for further analysis in Sessions 1 and 2. The actual data from Sessions 1 and 2 were averaged over 1.5 s in the middle of the time period (4 s) where steady-state values of force and moment were observed, and these average data were used to apply the ANIO method. First, principal component analysis (PCA) was performed on the steady-state finger force data across all the trials within each session separately (veridical, $2F_I$, and $0.5F_I$). In all subjects and three conditions, the amount of variance explained by the first two principal components (PCs) was over 90% (see *Results* section); hence, we assumed that experimental observations were confined to a two-dimensional plane in the four-dimensional force space for all three conditions. This indicates that the cost functions could feasibly be quadratic, which follows the Lagrange principle for the inverse optimization problem (Terekhov et al., 2010).

$$J = \frac{1}{2} \sum_i k_i (F_i)^2 + \sum_i (w_i) F_i \quad (1)$$

in which $i = \{\textit{index, middle, ring, and little}\}$. For the computational details of the ANIO method, see Terekhov et al. (2010) and Park et al. (2010).

The quadratic coefficient for the index finger, k_{index} was set at 1 across all conditions (see Park et al., 2010). If all the coefficients at the second-order terms are positive, the function complies with the assumption of the objective function minimization (Terekhov et al., 2010). Further, the dihedral angle between the plane of optimal solutions (obtained using the computed cost function for the same sets of task constraints) and the plane determined by the experimental observations was computed in order to quantify how well the cost-function from the ANIO predicted the experimental observation (smaller angle = better prediction). Further, the changes in the values of the second- and the first-order coefficients ($G2$ and $G1$, respectively) were computed in order to quantify the changes of the ANIO coefficients with changes in the index finger force gain as compared to the veridical condition:

$$G1_j^i = w_j^i - w_o^i \quad (2)$$

$$G2_j^i = k_j^i - k_o^i \quad (3)$$

in which $j = \{2F_I, 0.5F_I\}$, and k_o are the coefficients from the veridical feedback condition. Note that $G2^{index}$ is always 0 because the second-order coefficients for the index finger force were always set at 1.

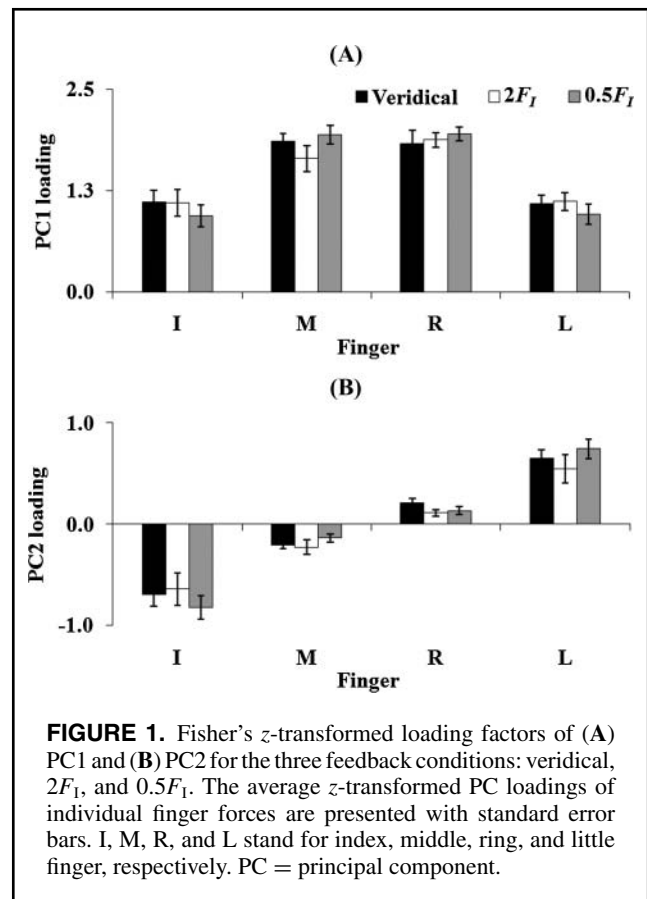
Statistics

Repeated measures analyses of variance (ANOVAs) with the factors Fingers and Condition were used to explore (a) how the dihedral angle between the optimal plane and the data plane was affected by the different feedback conditions (one-way repeated measures ANOVA) and (b) how the loadings of finger forces within each PC, $G1$, and $G2$ values were affected for different fingers by the manipulation of the feedback. Tukey's honestly significant difference tests and pairwise contrasts were used to explore significant effects at $p < .05$.

Results

Principal Component Analysis Results

The principal component analysis (PCA) was performed on each set of 81 observations (finger forces for the $\{F_{TOT}; M_{TOT}\}$ combinations) in each of the three feedback conditions (veridical, $2F_I$, and $0.5F_I$) and each subject separately. The first two PCs accounted for over 90% of the total variance in the finger force space for each of the three feedback conditions, and there was no significant difference among the conditions. Hence, the experimental observations were always confined to a two-dimensional plane in the four-dimensional force space. The loadings of all four finger forces in PC1 were large (>0.7) with the same sign (Figure 1A). In PC2, the loadings of the index and little finger



forces were larger than those of the middle and ring finger forces for all three conditions (Figure 1B). Also, the signs of the loadings for the index and middle finger forces were opposite to those for the ring and little finger forces. A two-way ANOVA (Condition \times Fingers) on Fisher z-transformed PC loadings showed only effects of fingers, PC1: $F(3, 18) = 33.87, p < .001$; PC2: $F(3, 18) = 62.95, p < .001$, but no significant differences across the three conditions and no Condition \times Fingers interactions.

ANIO Results

Application of the ANIO method allowed reconstructing quadratic cost functions with linear terms for each subject. To check the goodness of fit, the reconstructed cost functions were used to generate artificial data sets for each subject and condition; each data set formed a plane (plane of optimal solutions). The dihedral angle was defined as the angle between the planes of optimal solutions and the plane spanned by the two PCs based on the experimental observation. The dihedral angle for the veridical condition was $1.56^\circ \pm 1.77^\circ$, which was significantly smaller than in the other two conditions ($2F_I$: $4.57^\circ \pm 3.28^\circ$; $0.5F_I$: $5.42^\circ \pm 4.30^\circ$). A one-way ANOVA confirmed the main effect of condition, $F(2, 12) = 4.25, p < .05$.

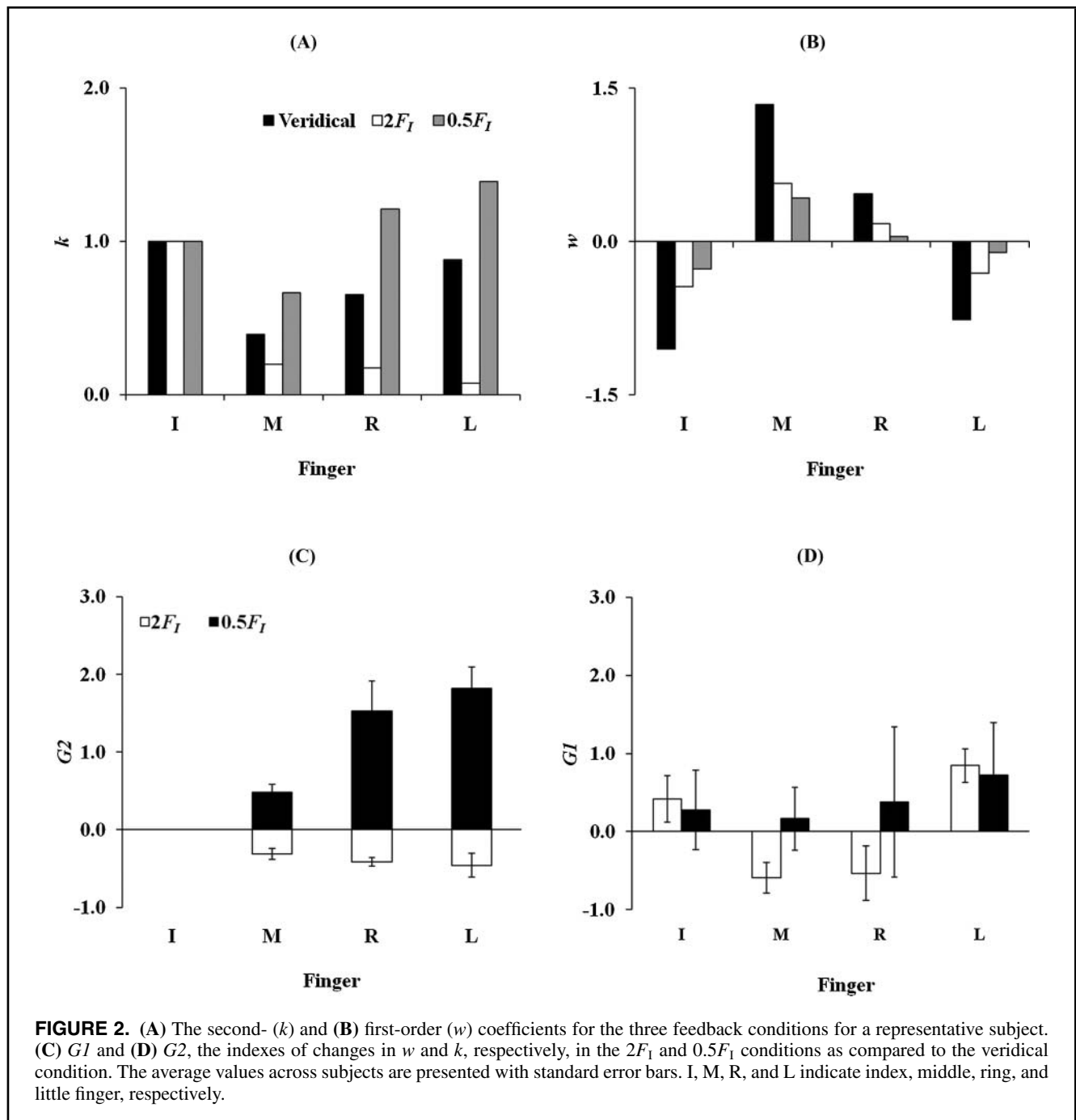


FIGURE 2. (A) The second- (k) and (B) first-order (w) coefficients for the three feedback conditions for a representative subject. (C) $G1$ and (D) $G2$, the indexes of changes in w and k , respectively, in the $2F_1$ and $0.5F_1$ conditions as compared to the veridical condition. The average values across subjects are presented with standard error bars. I, M, R, and L indicate index, middle, ring, and little finger, respectively.

Figures 2A and 2B show a typical pattern of the second- (k) and first-order (w) coefficients for the three conditions for a representative subject. Note that k values for all the fingers were always positive; k magnitudes for the middle, ring, and little fingers decreased in the $2F_1$ condition and increased in the $0.5F_1$ condition (Figure 2A). The w coefficients decreased in magnitude for all the fingers under the $2F_1$ and $0.5F_1$ conditions.

Figures 2C and 2D show the indices of changes in w and k ($G1$ and $G2$; see *Method* section) in the $2F_1$ and $0.5F_1$

conditions compared to the veridical condition. By definition, for the index finger $G2 = 0$. For the other three fingers, $G2$ were all positive for the $2F_1$ condition, whereas they were all negative for the $0.5F_1$ condition. In addition, $G2_{2F_1}^{ring}$ and $G2_{2F_1}^{little}$ were significantly larger than $G2_{2F_1}^{middle}$ (Figure 2C). These results were supported by a two-way ANOVA on $G2$ that showed significant main effects of Condition, $F(1, 6) = 43.99, p < .005$, and Fingers (3 levels: middle, ring, and little fingers), $F(2, 12) = 8.43, p < .01$, with a significant Condition \times Fingers interaction, $F(2, 12) = 9.26$,

$p < .005$). The interaction reflected the fact that $G2$ values differed among fingers under the $2F_1$ condition but not $0.5F_1$. There were no significant effects in a similar ANOVA run on $G1$, likely due to the large standard deviations of $G1$ under the $0.5F_1$ condition (Figure 2D). A one-way ANOVA on $G1$ for the $2F_1$ condition with fingers as the factor (four levels: I, M, R, and L) showed a significant main effect, $F(3, 18) = 5.17$, $p < .05$. The pairwise comparisons confirmed $G1_L > G1_M$. No such effects were seen for the $0.5F_1$ condition.

Discussion

Recent approaches to the problem of motor redundancy form two groups. One group focuses on the variability of solutions typical of natural, multielement tasks and explores patterns of covariation among elemental variables across repetitive trials. Among major recent developments within this group are the uncontrolled manifold (UCM) hypothesis (Scholz & Schöner, 1999) and the notion of synergies (Latash, 2010; Latash, Scholz, & Schöner, 2007). The other group of approaches focuses on patterns of involvement of elemental variables that are consistent across repetitive trials. Optimization is one of the commonly used techniques within this group (reviewed in Prilutsky, 2000). The two approaches emphasize two facets of the problem of motor redundancy: (a) what solution to select and (b) how to make this solution stable against unavoidable intrinsic and extrinsic perturbations. We recently used techniques from both groups (ANIO and UCM analyses) to analyze the same multifinger pressing task and suggested that the ideas of optimization and of synergies are not contradictory but complementary (Park et al., 2010). Note that one of the approaches, the UCM-based synergy analysis, involves analysis of elemental variables during repetitive trials to the same target, whereas the other approach (optimization, in particular ANIO) is based on analysis of trials to different targets.

The present study used the recently introduced ANIO method (Terekhov et al., 2010), which allows reconstructing cost functions based on a set of experimental observations of a redundant system. The method was used previously (Park et al., 2010; Terekhov et al.) and resulted in consistent across-subjects forms of the cost function (see Equation 1 in *Method* section). No interpretation of the function was offered in those studies. The main purpose of our study was to check a particular, biomechanical interpretation of the cost function using a manipulation originally suggested by Latash et al. (1998): making one finger artificially stronger or weaker. Based on that study, we informed the subjects on the manipulation of the feedback and gave them ample practice to get used to the new conditions.

Overall, the results support applicability of the ANIO method as well as using a quadratic cost function with linear terms for the multifinger pressing tasks. First, in all subjects and all conditions, the data were confined to a plane (PCA

results). This result, in combination with the Lagrange principle, allowed using the functional form of the cost function presented in Equation 1. The loading factors at the first two PCs did not differ significantly across conditions, which suggests that the data distributions were confined to similarly oriented planes. Second, using the cost function for direct optimization matched the data well: The two planes, optimal and experimental, were close to being parallel. Although the dihedral angle increased in the nonveridical conditions, its values were still relatively low (on average, about 5°). The increase in the dihedral angle could result from the unusual nature of the nonveridical conditions. Hence, on the whole we may confirm that the ANIO method and its results are robust with respect to the distorted visual feedback, which confirms our first and second hypotheses.

The offered interpretation of the cost function has been mainly supported by the data. Indeed, in the $2F_1$ condition, using the artificially stronger finger with the large lever arm to produce large forces would result in large moment of force values. Because the M_{TOT} range was limited (the same for low and high F_{TOT}), this is a suboptimal strategy. This was reflected in relatively low coefficients at the second-order terms (k) for the other three fingers. Note that a lower coefficient in the cost function implies that using high values of force by that particular finger is relatively less costly (encouraged). An opposite trend was seen in the $0.5F_1$ condition. With respect to the coefficients at the first-order terms (w), the results are more ambiguous. At low F_{TOT} , when the first-order terms are expected to dominate, producing the same range of M_{TOT} is facilitated by using finger forces with larger lever arms. As a result, w values for the index and little fingers are negative (encouraged). Making the index finger stronger is expected to scale down its involvement (for the same M_{TOT} values); higher w values were indeed observed in the experiments. There was a similar increase in w for the little finger, whose force was used veridically. We can offer only a tentative interpretation assuming that it was due to the symmetrical involvement of the two fingers with the largest lever arms. Thus, the third and fourth hypotheses have also been supported by the results.

In redundant motor tasks, theoretically, very large subspaces in the space of elemental variables are available, limited by anatomical and physiological constraints, which are equally able to solve the tasks. However, in each particular task humans use only a relatively small region within this potentially available subspace. Several approaches linked this choice to stability of solutions, particularly with respect to possible unavoidable variations in the magnitudes of elemental variables (Müller & Sternad, 2004; Scholz & Schöner, 1999). Our approach is different. We do not single out a priori a specific feature of performance (e.g., stability, energy expenditure, fatigue) and assume that this feature dictates the choice of an area from which solutions are selected. An objective mathematical method is used to produce a description of this area (in terms of a cost function) based on actual

performance. We see this as a major advantage of the ANIO approach.

The main theorem of the ANIO approach, the theorem of uniqueness, is formulated for an infinite number of precise experimental points. In experiments, the data cover a limited subspace and are also affected by noise. Given these two factors, the ANIO method can only ensure that the computed cost function is close to the true one (assuming that a true one exists). In all subjects and all conditions, the data tended to form a plane, but they deviated from a perfect plane. As a consequence, it cannot be claimed that the cost function is quadratic, only that it can be well approximated by a quadratic cost function.

Consider the following example (suggested by an anonymous reviewer). Imagine that a similar set of four-finger tasks is performed by a hypothetical controller that tries to minimize a cost function representing the sum of finger forces cubed, $J_1 = \sum_i f_i^3$. If the same set of constraints as in our experiment is applied and an optimal data set is defined, PCA on that surrogate data set shows that 99.99% of the data are explained by the first two PCs. Thus, high planarity of the data by itself does not ensure that the true cost function is quadratic. However, this result implies that the cost function can be well approximated by a quadratic function within the range of available data. Figure 3 shows the values of two cost functions, J (as in our study) and J_1 , over a range of forces. For a given force magnitude, values for nine different moment of force magnitudes are shown; they do not differ by much. Note that the computed points for the two analyses and the two curves are nearly on top of each other. Practically, over the studied ranges of finger forces the values of the two cost functions look indistinguishable. So, J may not be the true function but it approximates the true function very closely.

Over the studied ranges of finger forces, many cost functions can potentially produce planar distributions of computed values. However, some of these planes coincide with the experimentally observed one, whereas others do not. To illustrate this point, we applied the mentioned cubic cost function to compute the plane of optimal solutions for our task constraints and computed the dihedral angle between the plane of optimal solutions and the plane of actual observations for each subject. For the cubic function the angle was $20.89^\circ \pm 8.57^\circ$, whereas it was only $1.56^\circ \pm 1.77^\circ$ for the quadratic function defined by the ANIO method. Another metric that can be used to compare different cost functions is the root mean square deviation of the computed values from the observed ones. Recently, we compared several traditional cost functions to the ones generated by the ANIO approach in their ability to fit experimentally observed data sets for a similar force-moment production task. The ANIO approach generated cost functions with significantly better fit to the experimental data as compared to the traditional cost functions (unpublished).

An important lesson from this example (see also Terekhov et al., 2010; Terekhov & Zatsiorsky, 2011) is that—due to

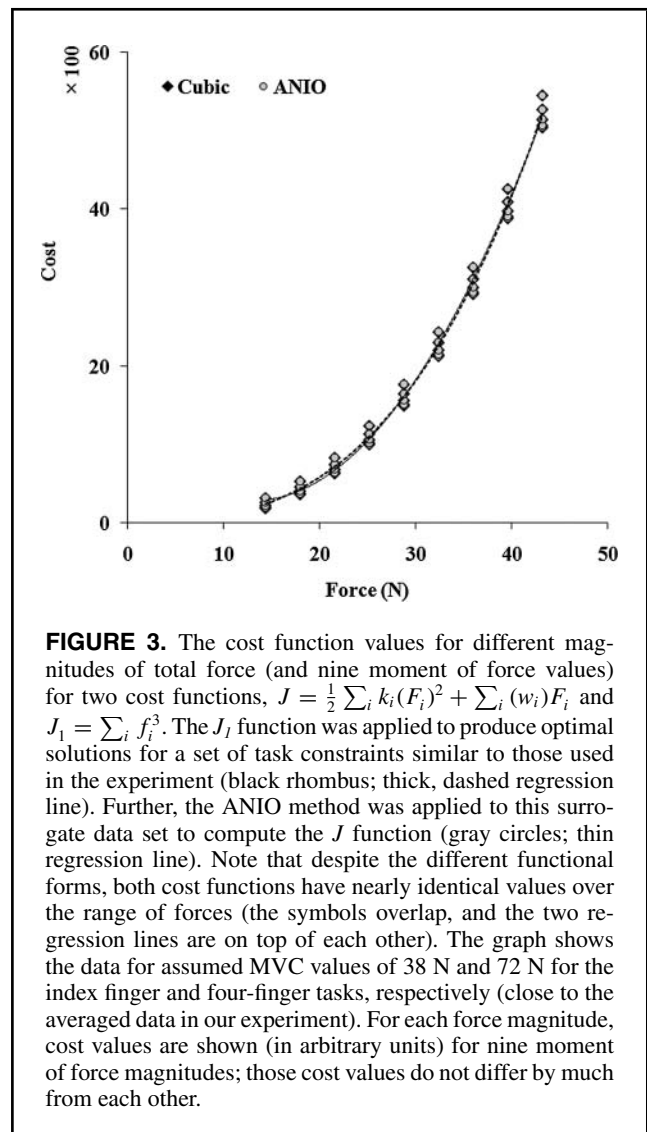


FIGURE 3. The cost function values for different magnitudes of total force (and nine moment of force values) for two cost functions, $J = \frac{1}{2} \sum_i k_i (F_i)^2 + \sum_i (w_i) F_i$ and $J_1 = \sum_i f_i^3$. The J_1 function was applied to produce optimal solutions for a set of task constraints similar to those used in the experiment (black rhombus; thick, dashed regression line). Further, the ANIO method was applied to this surrogate data set to compute the J function (gray circles; thin regression line). Note that despite the different functional forms, both cost functions have nearly identical values over the range of forces (the symbols overlap, and the two regression lines are on top of each other). The graph shows the data for assumed MVC values of 38 N and 72 N for the index finger and four-finger tasks, respectively (close to the averaged data in our experiment). For each force magnitude, cost values are shown (in arbitrary units) for nine moment of force magnitudes; those cost values do not differ by much from each other.

the limited range of data and nonzero noise—the ANIO method applied to data sets is not expected to produce the true cost function, only a function that approximates the true one closely. This should not be viewed as a major flaw of the approach but rather as a limitation inherent to the idea of optimization. Indeed, most researchers would probably agree that motor performance is not defined by a single cost function applicable across tasks and ranges of performance variables, and that most cost functions considered in previous studies are limited in their ability to describe the data precisely (see reviews in Prilutsky, 2000; Rosenbaum et al., 2001).

We are presently only making the first steps toward understanding the mechanical (and potentially physiological and psychological) meaning of the cost functions reconstructed using the ANIO method. This study makes us optimistic with respect to further development and applications of this method. The hypotheses formulated were supported

statistically and the overall pattern of the results is consistent with the offered interpretation of the cost function. However, the large variability across subjects suggests that mechanics is not the only factor that defines patterns of finger coordination in such tasks. Our next plans are to explore sensitivity of the ANIO method to changes in finger coordination that happen with more physiological changes such as those that accompany fatigue and healthy aging (cf. Kapur, Zatsiorsky, & Latash, 2010; Singh, SKM, Zatsiorsky, & Latash, 2010).

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